2103000206020033

EXAMINATION FEBRUARY-MARCH 2024
BACHELOR OF SCIENCE (SIXTH SEMESTER)
MATHEMATICS-VIII (MTH-603-REAL ANALYSIS-III) - LEVEL 2

[**Time: As per schedule]** **[Max. Marks: 50]**

Instructions:
1. **1. Fill up strictly the following details on your answer book**
   a. Name of the Examination: **BACHELOR OF SCIENCE (SIXTH SEMESTER)**
   b. Name of the Subject: **MATHEMATICS-VIII (MTH-603-REAL ANALYSIS-III)**
   c. Subject Code No: **2103000206020033**

2. Sketch neat and labelled diagram wherever necessary.
3. Figures to the right indicate full marks of the question.
4. All questions are compulsory.
5. Follow usual notations.

Seat No:

Student's Signature

Q.1 Do as directed (any five): (2 marks each) **10**

1. Find σ_n for the sequence $S_n = (-1)^n$.

2. Give an illustration of $(C,2)$ summable sequence which is not $(C, 1)$ summable.

3. Define uniform convergence for the sequence of the real valued functions.

4. Evaluate: $\lim_{n\to\infty} \int_0^1 f_n(x)\, dx$, where $f_n(x) = nx,(1 - x^2)^n;\ 0 \leq x \leq 1$.

5. Prove that a singleton set is of measure zero.

6. Define Upper Riemann Integral and Lower Riemann Integral for a bounded function f on $[a,b]$.

7. If $f(x) = \int_0^x \sqrt{t + t^6} dt (x > 0)$ then find $f'(1)$.

8. If f is continuous on [a, b], then prove that there exists c $\in (a, b)$ such that $\int_a^b f(x)\,dx = f(c)(b-a)$.

Q.2 Attempt any two:(5 marks each) 10

1. Let $\sum_{n=1}^{\infty} a_n$ be $(C, 1)$ summable and let $t_n = a_1 + 2a_2 + 3a_3 + \_\_ + na_n$.
   If $\lim\limits_{n\to\infty} \frac{t_n}{n} = 0$, then prove that the series $\sum_{n=1}^{\infty} a_n$ converges.

2. Prove that the sequence -1,2,2,-1,2,2,-1,______ is $(C, 1)$ summable.

3. Prove that the series $\sum_{n=1}^{\infty}(-1)^n$ is $(C, 1)$ summable.

Q.3 Attempt any two:(5 marks each) 10

1. Prove that the sequence of functions $\{f_n\}_{n=1}^{\infty}$ converges uniformly to f on E if and only if $\underset{x\epsilon E}{lub}\ |f_n(x) - f(x)| \to 0$ as $n \to \infty$.

2. Let $f_n(x) = \frac{x}{1+nx}$; $0 \le x < \infty$.
   Prove that for given $\epsilon > 0$ we can find $N \epsilon I$ such that
   $|f_n(x) - f(x)| < \epsilon; n \ge N$ holds simultaneously for all $x \epsilon [0, \infty)$,
   where $f(x) = 0; 0 \le x < \infty$.

3. Let X_n be the characteristic function of the open interval $(0, \frac{1}{n})$
   Let $f_n(x) = nX_n(x); 0 \le x \le 1$. Prove that $\lim\limits_{n\to\infty} \int_0^1 f_n(x)dx \neq \int_0^1 \lim\limits_{n\to\infty} f_n(x)dx$

Q.4 Attempt any two:(5 marks each) 10

1. If $f \epsilon R[a, b]$ and a < c < b, then prove that $f \in R[a, c], f \in R[c, b]$ *and* $\int_a^b f = \int_a^c f + \int_c^b f$.

2. Prove that continuous function on the closed bounded interval $[a, b]$ is Riemann Integrable.

3. If A is not of measure zero, if $B \subset A$, and if B is of measure zero then prove that $A - B$ is not of measure zero. Use it to prove that the set of all irrational numbers is not of measure zero.

Q.5 Attempt any two:(5 marks each) **10**

1. If f is a continuous on the closed bounded interval $[a, b]$ and if $F(x) = \int_a^x f(t)dt (a \leq x \leq b)$, then prove that $F'(x) = f(x) (a \leq x \leq b)$.

2. If f, f' and g' are continuous on [a, b], then prove that
   i) there exist $c \epsilon$ (a, b) such that $\int_a^b f(x) = f(c)(b - a)$.
   ii) $\int_a^b f(x)\, g'(x)dx = f(b)g(b) - f(a)g(a) - \int_a^b f'(x)g(x)dx$.

3. If $f \epsilon\ R$[a, b], then prove that $|f| \epsilon$ R[a, b] and $\left|\int_a^b f\right| \leq \int_a^b |f|$.

*****